

WOLFRAM ALPHA AS AN ON-LINE TEACHING TOOL FOR ORDINARY DIFFERENTIAL EQUATIONS SOLVING

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Abstract

Wolfram Alpha is known as a unique web knowledge tool that offers search and visualization of information and it is oriented mainly to science. There are many possibilities how Wolfram Alpha can be used. We have focused on the fact that Wolfram Alpha can be applied also as a tool in math learning process. It is exemplified by the theme of differential equations solving. Some ideas for an innovative approach in educational methods supported by Wolfram Alpha are presented in this contribution.

Key words: Wolfram Alpha, solving of ordinary differential equations, ODE solving, online advanced calculator, answer engine, e-learning, self-instructed study.

WOLFRAM ALPHA JAKO VÝUKOVÝ NÁSTROJ PŘI ŘEŠENÍ OBYČEJNÝCH DIFERENCIÁLNÍCH ROVNIC

Resumé

Wolfram Alpha je označován jako unikátní výpočetní znalostní webový nástroj, který je orientován na vyhledávání a vizualizaci informací souvisejících především s vědeckými oblastmi. Z mnoha možností, jak může být využíván, jsme se zaměřili na potenciál jeho zařazení do vyučování matematiky. V příspěvku uvádíme několik myšlenek, jak použít Wolfram Alpha při výuce řešení diferenciálních rovnic.

Klíčová slova: Wolfram Alpha, obyčejné diferenciální rovnice, online vědecká kalkulačka, e-learning, samostudium.

Introduction

The teaching of mathematics at universities is usually supported by many kinds of free or commercial mathematical software either requiring its installation or running online nowadays. Considering the internet connection is quite common at school, the progress in web technologies brings every day new ways how to do teaching more interesting and effective as well. One of free online tools that could be useful not only for scientists but also for teachers and students is computational knowledge machine Wolfram Alpha.

1 Some facts about Wolfram Alpha

Wolfram Alpha is an online answer engine developed with webMathematica technology by Wolfram Research that answers factual queries directly by computing it from structured data. Because Wolfram Alpha is built on core of Mathematica, it is very useful for answers to such a type of questions, where the computational force of Mathematica can be used.

From the time when Wolfram Alpha was appeared in May 2009 on web, many people have a tendency to compare it with the search engine of Google. But the main vision of this project was and is completely different, to use web search engine together with all known models, computational methods and algorithms with the aim to answer more precisely. This is the reason the project is of a long-term character and it is in progress all the time.

2 Wolfram Alpha in math education

In the context of using Wolfram Alpha possibilities in education of mathematics, we could see a few areas, where Wolfram Alpha offers tools for being the math study more effective. It can be applied e. g.:

- in preparation of study materials, tests and exercises to check the solution
- as a rapid tool to obtain the numerically correct result and plots giving an additional representation
- in self-study for students.

To be online and use web services (such as Google, Facebook, Twitter...) is absolutely natural for most of students, so one can expect the work with another type of search engine as the Wolfram Alpha will not be difficult for them. Unfortunately a small problem concerning of the question creation appears here. It is very important to put the question specifically and correctly.

3 Wolfram Alpha in solving of differential equations (ODEs)

We have illustrated the Wolfram Alpha support of the theme of ordinary differential equations solving because recently (in January 2012) an entirely new helpful functionality - “Step-by-step“ math, relating to differential equations solving was added. Another reason is that differential equations are fundamental to many fields and this part of mathematics seems to be rather hard in many students opinion and this support can be helpful. The following example (Fig. 1, Fig. 2) shows how Wolfram Alpha copes with solving ODE. The benefit of rapid and easy obtained result is conditional on the question form. In case of computational queries syntax expected in the input window is Mathematica syntax, so it is similar to most of CAS systems.

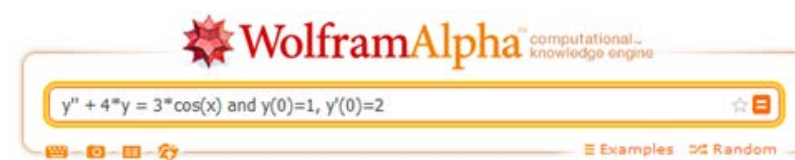


Fig 1. The form of input data

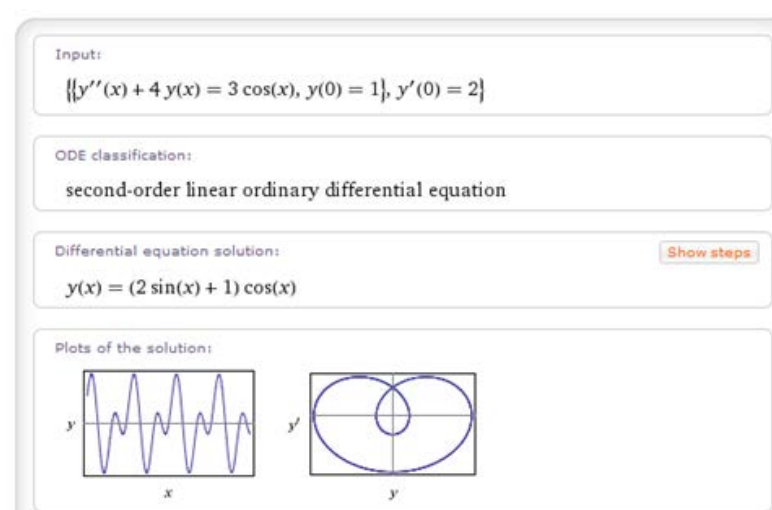
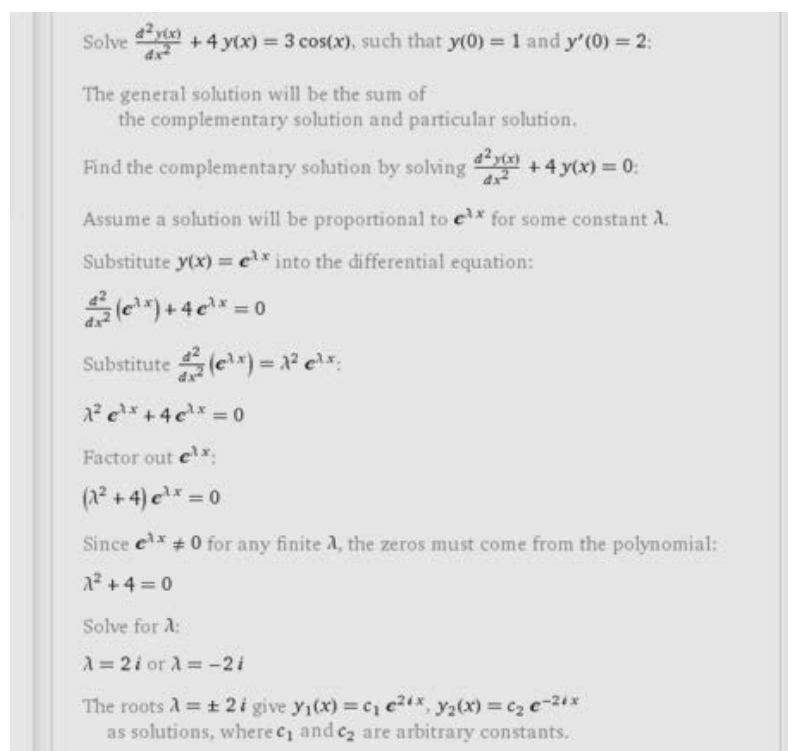


Fig. 2. An answer of Wolfram Alpha is the solution of ODE

Besides of the computing the solution, we can see ODE classification and visualization of the solution result (when it is possible). In this case the integral curve has been displayed at the end of the output.

The special feature is the explicit and very detailed description of the solution displayed step-by-step with complete explanation, appearing after click to “show steps“ (Fig. 3). This functionality is the added value, which should be appreciated mainly by students during their self-study.



Solve $\frac{d^2 y(x)}{dx^2} + 4 y(x) = 3 \cos(x)$, such that $y(0) = 1$ and $y'(0) = 2$;

The general solution will be the sum of the complementary solution and particular solution.

Find the complementary solution by solving $\frac{d^2 y(x)}{dx^2} + 4 y(x) = 0$;

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ .

Substitute $y(x) = e^{\lambda x}$ into the differential equation:

$$\frac{d^2}{dx^2}(e^{\lambda x}) + 4 e^{\lambda x} = 0$$

Substitute $\frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x}$;

$$\lambda^2 e^{\lambda x} + 4 e^{\lambda x} = 0$$

Factor out $e^{\lambda x}$;

$$(\lambda^2 + 4) e^{\lambda x} = 0$$

Since $e^{\lambda x} \neq 0$ for any finite λ , the zeros must come from the polynomial:

$$\lambda^2 + 4 = 0$$

Solve for λ ;

$$\lambda = 2i \text{ or } \lambda = -2i$$

The roots $\lambda = \pm 2i$ give $y_1(x) = c_1 e^{2ix}$, $y_2(x) = c_2 e^{-2ix}$ as solutions, where c_1 and c_2 are arbitrary constants.

Fig. 3. A part of the differential equation solution step-by-step

This step-by-step program has the ability to solve a lot of types of first-order equations (separable, linear, Bernoulli, homogeneous and exact) and higher-order equations as well, using method of undetermined coefficients, variation of constants, the method of Laplace transforms, and many more.

Conclusion

Due to the advanced computational and search potential Wolfram Alpha as a free online tool could be an excellent help for teachers and students as well. The detailed step-by-step performance of the solution and its graphic representation is great for understanding in process of self-study. However in spite of many advantages mentioned above it is necessary to ensure that support by computational tools should be the only next step in learning after obtaining basic skills and it is just teacher who holds full responsibility for it.

Literature

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