MS EXCEL UTILISATION WITH DIFFERENTIAL EQUATIONS APPLICATIONS

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Abstract

The contribution presents the possibilities of MS Excel application to mathematics education at technical universities, specifically to applicable tasks related to the theory of differential equations.

Key words: differential equations, applications, numerical mathematics, MS Excel

1 Introduction

Mathematics education at faculties of technical universities requires that students are taught how to apply mathematics to technology, how to become skillful in modeling and numerical solution of predominantly technical problems. That is why teachers of mathematics at that type of faculties should naturally make every effort to deepen an applicable character of mathematical subjects through the implementation of technical applications to the teaching, to make the process of education more effective by the utilisation of IKT and introduction of new teaching forms. The intention of the article is to present the possibilities of MS Excel utilisation with applications of a selected type of differential equations, thus contributing to the above mentioned aim.

2 Design of Mathematical Model of the Given Problem in the Form of Differential Equation

In order to increase the motivation level and to eliminate the absence of applicable tasks, we recommend to solve the following task (Fulier, 2001) at practical classes of mathematical analysis or the so called computing practical classes of numerical mathematics:

When blowing through the forest, wind loses its velocity due to the resistance of the trees. It was validated experimentally that the loss of a wind velocity is proportional to the length of this path and the magnitude of a wind velocity. Find the wind velocity heading to inwards the forest at a distance of 150 m from the edge of the forest, if the wind initial velocity at the edge of the forest was $v_0 = 12\,\mathrm{ms}^{-1}$ *and at a distance of* $10\,\mathrm{m}$ *from the edge of the forest the wind velocity reduced to the value* $v_1 = 9{,}85\,\mathrm{ms}^{-1}$.

With the application of the theory of differential equations to the above presented problem and with the design of a mathematical model we have to consider that this refers to a proportionally retarded motion and according to the physical laws it holds : $dv = -k\,.v\;ds$, where $v = v(s)$ is the wind velocity at a distance s from the edge of the forest. A mathematical model of this applicable task is an ordinary differential equation.

$$\frac{dv}{ds} = -k\,.v \tag{1}$$

with the initial condition $v(0)=12$ and with a supplementary condition $v(10)=9{,}85$. It is evident that this refers to a separable differential equation, hence we will obtain a universal solution in the form $v=C.e^{-ks}$. Utilising the initial condition we will determine the value of a arbitrary constant $C=12$. Using a supplementary condition $v(10)=9{,}85$ we will determine a constant k from the equation $9{,}85=12.e^{-10k} \Rightarrow k=\frac{1}{10}.ln\frac{12}{9{,}85}$. Taking into consideration (1) and the constant k, for the given situation we will obtain a differential equation of the wind velocity reduction in the form:

$$\frac{dv}{ds}=\frac{1}{10}\ln\frac{9{,}85}{12}.v \qquad (2)$$

whence by the variables separation and the utilisation of the initial condition we will obtain the solution to the equation:

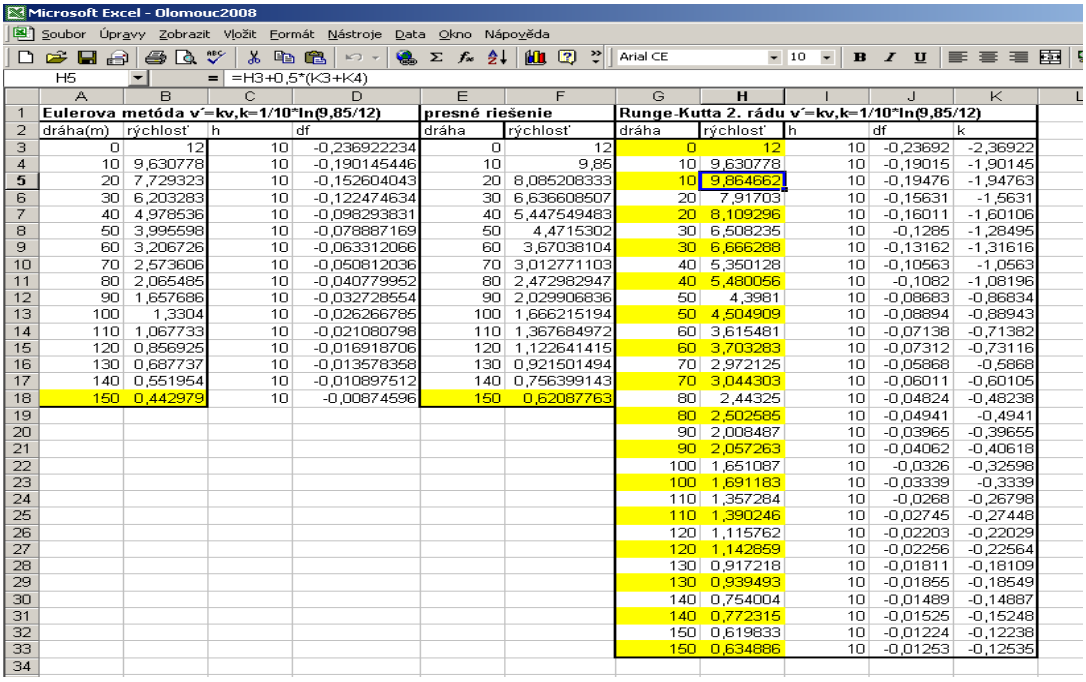
$$v=12.\left(\frac{9{,}85}{12}\right)^{\frac{s}{10}} \qquad (3)$$

It is evident that the same solution can be obtained by the constants k and C substitution for a universal solution. As we are interested in the wind velocity at the distance of 150 m from the edge of the forest, it is enough to substitute $s=150\,\text{m}$ in the relation (3) and we obtain that the wind velocity at this distance is only $v=0{,}62087763\ \text{ms}^{-1}$.

3 Numerical Solution with MS Excel Utilisation

Let us illustrate how a discrete solution to the differential equation (2) with the initial condition $v(0)=12$ can be found by numerical methods. A numerical solution to the equation (2) will not be obtained in the form of the function $v=v(s)$ which expresses the wind velocity reduction, we will obtain only its approximation in the form of couples $(s_i),(v_i)$ presented in Excel in a column form. For an approximate solution, Euler's method and the 2nd order Runge-Kutta method with the step $h=10$ will be used. During the problem solving it will follow that the equation (2) requires the solution within the interval in metres $\langle 0{,}150\rangle$ expressing a wind distance from the forest edge. According to the algorithm of Euler's method of the differential equation $v'(s)=f(s,v)$ solution within the interval $\langle s_0, s_0+n.h\rangle$ with the step h and the initial condition $v(s_0)=v_0$ we write into Excel cells the required relations, see fig.1. In column A we will create a numerical sequence expressing a change of the wind distance from the forest edge starting with the distance of 0 m. In the cell B3, the initial condition is used. In D3 there is a record of the right side of the differential equation (2) in the form $v'(s)=\frac{1}{10}ln\frac{9{,}85}{12}.v$, which after being rewritten into Excel looks as follows: =(1/10)*LN(9,85/12)*B3. Velocity v_1 in the distance $s_1=10\,\text{m}$ is expressed according to the algorithm of Euler‘s method $v_1=s_0+h.v'(s_0)=s_0+h.f(s_0,v_0)$ in the cell B4 by the formula: =B3+C3*D3. The cell D4 will be completed by copying the cell D3. Other approximated values v_i of velocity v will be obtained by copying the line 4. In columns

E, F there is expressed an exact solution obtained analytically. Algorithm of the solution of the differential equation $v'(s) = f(s, v)$ within the interval $\langle s_0, s_0 + n.h \rangle$ with the step h and the initial condition $v(s_0) = v_0$ according to the 2nd order Runge-Kutta method can be expressed in a simplified form that can be seen in table 1. If we utilise the relations of the mentioned algorithm and rewrite them into Excel cells, we will obtain a discrete solution to the equation (2), i.e. approximated values of velocity v_i at the distance s_i. In fig. 1 they are pointed up in columns G, H; because of the lucidity, they are not plotted in graph in fig.2. Comparing the approximated values v_i at the distance s_i to the accurate values obtained analytically we can see that Euler's method provides less accurate results.



Microsoft Excel - Olomouc2008

Soubor Úpravy Zobrazit Vložit Formát Nástroje Data Okno Nápověda

H5 = =H3+0,5*(K3+K4)

	A	B	C	D	E	F	G	H	I	J	K
1	Eulerova metóda v´=kv,k=1/10*ln(9,85/12)				presné riešenie		Runge-Kutta 2. rádu v´=kv,k=1/10*ln(9,85/12)				
2	dráha(m)	rýchlosť	h	df	dráha	rýchlosť	dráha	rýchlosť	h	df	k
3	0	12	10	-0,236922234	0	12	0	12	10	-0,23692	-2,36922
4	10	9,630778	10	-0,190145446	10	9,85	10	9,630778	10	-0,19015	-1,90145
5	20	7,729323	10	-0,152604043	20	8,085208333	10	9,864662	10	-0,19476	-1,94763
6	30	6,203283	10	-0,122474634	30	6,636608507	20	7,91703	10	-0,15631	-1,5631
7	40	4,978536	10	-0,098293831	40	5,447549483	20	8,109296	10	-0,16011	-1,60106
8	50	3,995598	10	-0,078887169	50	4,4715302	30	6,508235	10	-0,1285	-1,28495
9	60	3,206726	10	-0,063312066	60	3,67038104	30	6,666288	10	-0,13162	-1,31616
10	70	2,573606	10	-0,050812036	70	3,012771103	40	5,350128	10	-0,10563	-1,0563
11	80	2,065485	10	-0,040779952	80	2,472982947	40	5,480056	10	-0,1082	-1,08196
12	90	1,657686	10	-0,032728554	90	2,029906836	50	4,3981	10	-0,08683	-0,86834
13	100	1,3304	10	-0,026266785	100	1,666215194	50	4,504909	10	-0,08894	-0,88943
14	110	1,067733	10	-0,021080798	110	1,367684972	60	3,615481	10	-0,07138	-0,71382
15	120	0,856925	10	-0,016918706	120	1,122641415	60	3,703283	10	-0,07312	-0,73116
16	130	0,687737	10	-0,013578358	130	0,921501494	70	2,972125	10	-0,05868	-0,5868
17	140	0,551954	10	-0,010897512	140	0,756399143	70	3,044303	10	-0,06011	-0,60105
18	150	0,442979	10	-0,00874596	150	0,62087763	80	2,44325	10	-0,04824	-0,48238
19							80	2,502585	10	-0,04941	-0,4941
20							90	2,008487	10	-0,03965	-0,39655
21							90	2,057263	10	-0,04062	-0,40618
22							100	1,651087	10	-0,0326	-0,32598
23							100	1,691183	10	-0,03339	-0,3339
24							110	1,357284	10	-0,0268	-0,26798
25							110	1,390246	10	-0,02745	-0,27448
26							120	1,115762	10	-0,02203	-0,22029
27							120	1,142859	10	-0,02256	-0,22564
28							130	0,917218	10	-0,01811	-0,18109
29							130	0,939493	10	-0,01855	-0,18549
30							140	0,754004	10	-0,01489	-0,14887
31							140	0,772315	10	-0,01525	-0,15248
32							150	0,619833	10	-0,01224	-0,12238
33							150	0,634886	10	-0,01253	-0,12535
34											

Fig.1 *Numerical solution to the differential equation of the wind velocity reduction in Excel*

Table 1

s_i - path	v_i - velocity	k_i
s_0 $s_0 + h$	v_0 $v_0 + k_0$	$k_0 = h.f(s_0, v_0)$ $k_1 = h.f(s_0 + h, v_0 + k_0)$
s_1 $s_1 + h$	$v_1 = v_0 + 0{,}5.(k_0 + k_1)$ $v_1 + k_2$	$k_2 = h.f(s_1, v_1)$ $k_3 = h.f(s_1 + h, v_1 + k_2)$
s_2 $s_2 + h$	$v_2 = v_1 + 0{,}5.(k_2 + k_3)$ $v_2 + k_4$	$k_4 = h.f(s_2, v_2)$ $k_5 = h.f(s_2 + h, v_2 + k_4)$
...	...	...
s_n	$v_n = v_{n-1} + 0{,}5.(k_{2n-2} + k_{2n-1})$	

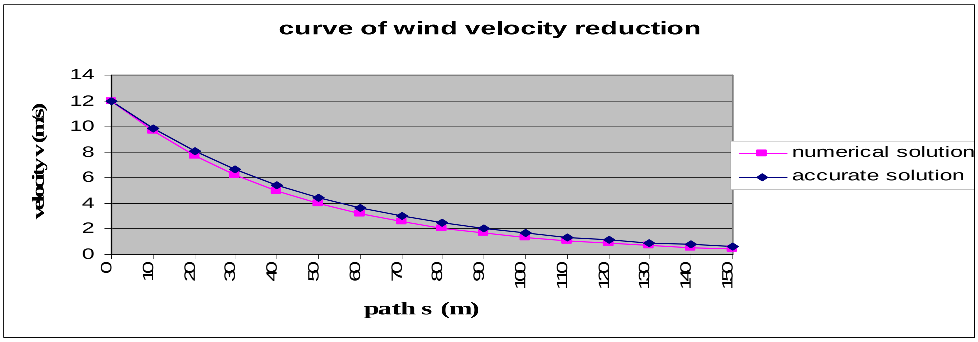


Fig. 2 Integral curve of the solution to the differential equation of the wind velocity reduction

4 Conclusion

When verifying the effectiveness of MS Excel utilisation as a software support with the applications of differential equations, for example within numerical mathematics teaching, MS Excel program in general proved as a suitable means. Its advantage consists in its availability as well as in the fact that students are very familiar with MS Excel environment. Most students are able to work with cells (copy, shift…), formulas (to record a function prescription), or to create graphs. Teaching it to the students at practical classes is not time consuming either. Based on experience, the following procedure has been approved - a classical explanation, a given task solution, verification if the students cope with the algorithm, utilization of a computing technique with a suitable mathematical software. ICT implementation to the numerical mathematics teaching enables not only to calculate more tasks, but also compare individual methods from the viewpoint of error estimation, promptness, convergence and suitability of the given method. Another undoubted advantage is a graphical interpretation of numerical solution results with a computing support. However, the goal remains that students cope with and master numerical methods to such an extent that they are able to program them independently in any suitable environment of a mathematical software.

5 Literature

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