ABOUT SOLUTIONS OF THE NONLINEAR DIFFERENTIAL SYSTEMS

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Abstract

In the article are defined some properties of the solutions systems differential equations.

Key words: Differential equations, properties of the solutions.

1 Introduction

Let the system

$$\begin{aligned} x_1'(t) &= g_1(t, x(t))x_1(t) \\ x_2'(t) &= g_2(t, x(t))x_2(t) \\ \dots\ \dots\ &\dots\ \dots\ \dots \\ x_n'(t) &= g_n(t, x(t))x_n(t) \end{aligned} \tag{1}$$

Is the system non-linear differential equations, where
$g_i(t, x_1(t), x_2(t), \dots, x_n(t)) \in C_0(D \equiv J \times R^n, R)$, $i \in \{1, 2, \dots, n\}$, C_0
Is the space of the real continuous functions with variables $t, x_1(t), x_2(t), \dots, x_n(t)$ defined on the set $J \times R^n$. We suppose that every solution
$x(t) = (x_1(t), x_2(t), \dots, x_n(t))$, $x_1(t_0) = x_1^0, x_2(t_0) = x_2^0, \dots, x_n(t_0) = x_n^0, \quad t_0 \in J$
exists on the interval J . We denote $h > t_0 > 0$ the right end point of the interval J and $J_0 = \langle t_0, h)$. If $D_0 \subset D$ is the open empty set and the derivatives

$$\begin{aligned} &\partial g_i(t, x_1(t), x_2(t), \dots, x_n(t)) / \partial x_1(t), \\ &\partial g_i(t, x_1(t), x_2(t), \dots, x_n(t)) / \partial x_2(t)\ , \\ &\dots \\ &\partial g_i(t, x_1(t), x_2(t), \dots, x_n(t)) / \partial x_n(t) \end{aligned}$$

are continuous functions in the set D for every $i \in \{1, 2, \dots, n\}$, then for every point $(t_0, x_1^0, x_2^0, \dots, x_n^0) \in D$ cross only one integral curve $x \in D$ of the system (1).

2 Properties of the solutions – trivial solutions, positive solution, bounded solution

Definition 1. The solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$, $t \in J_0$ of the system (1) called $x_i -$ trivial $i \in \{1, 2, \dots, n\}$ is the determined if $x_i(t) = 0$ on the all interval J_0 .
$x(t)$ is $x_i -$non-trivial solution in others cases.

The solution is non-trivial if at least for $x_i(t)$ of the system (1) is $x_i -$non-trivial, $i \in \{1, 2, \dots, n\}$.

If $x(t) = (0,0,\dots,0)$, $t \in J_0$ is $x_1, x_2, \dots, x_n -$trivial solution of the system (1), so $g_i(t,0,0,\dots,0) = 0$ for every $t \in J_0$,. $i \in \{1, 2, \dots, n\}$.

If $x(t)$, $t \in J_0$ is $x_1 -$ trivial ($x_n -$trivial) solution of the system (1) so

$$g_1(t,0,x_2(t),\dots,x_n(t)) = 0,$$
$$x_2'(t) = g_2(t,0,x_2(t),\dots,x_n(t)),$$
$$\dots$$
$$x_n'(t) = g_2(t,0,x_2(t),\dots,x_n(t)),$$
$$(\, x_1'(t) = g_1(t,x_1(t),x_2(t),\dots x_{n-1}(t),0),$$
$$x_2'(t) = g_2(t,,x_1(t),x_2(t),\dots,x_{n-1}(t),0),$$
$$\dots$$
$$g_n(t,x_1(t),x_2(t),\dots,x_{n-1}(t),0) = 0\,)$$

for all every $t \in J_0$.

If $J_1 \subset J_0$ is the interval and $x(t)$, $t \in J_1$ is $x_i -$trivial solution of the system (1), $i \in \{1, 2, \dots, n\}$ so $x(t)$ is $x_i -$trivial solution on the interval J_1.

We discriminate contradict situations for the position of the integral curve $x \in D$ of the system (1) if $x \in o_{tx_1}$ so $x_1 -$non-trivial, $x_2 -$trivial, $\dots, x_n -$trivial solution, if $x \in o_{tx_n}$ so $x_n -$non-trivial, $x_1 -$trivial, $x_2 -$trivial,…, $x_{n-1} -$trivial solution, if $x \in o_{tx_1 x_2 \dots x_n}$ so $x_1 -$non-trivial, $x_2 -$non-trivial,…, $x_n -$non-trivial solution.

If $x \in o_t$ thus $x_1 -$trivial, $x_2 -$trivial, …, $x_n -$trivial solution.

Definition 2. The solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$, $t \in J_0$ of the system (1) called $x_i(t) -$ positive ($x_i(t) -$negative) $i \in \{1, 2, \dots, n\}$, if $x_i(t)$ is the positive (negative) function on the interval J_0.

Definition 3. The solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$, $t \in J_0$ of the system (1) called $x_i -$bounded solution, $i \in \{1, 2, \dots, n\}$, if $x_i(t)$ is bounded function on the interval J_0. The solution $x(t)$ is $x_i -$non-bounded solution in other cases.

If $x_i(t)$ is form above non-bounded function so $x(t)$ called $x_i -$ from above non-bounded solution, $i \in \{1, 2, \dots, n\}$.

If $x_i(t)$ is from below function so $x(t)$ called $x_i -$from below non-bounded function, $i \in \{1, 2, \dots, n\}$.

The solution $x(t)$ is $x_i -$ from above non-bounded solution so we called $x_i -$from above non-bounded ($x_i -$from below non-bounded), $i \in \{1, 2, \dots, n\}$

4 Conclusion

The solution $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))$, $t\in J_0$ of the system (1) called constant, if $x_1(t_0)=x_1^0, x_2(t_0)=x_2^0,\ldots, x_n(t_0)=x_n^0$, $t_0\in J$ on the interval J_0. The solution $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))$ is non-constant solution in other cases.

If $J_1\subset J_0$ is the interval and $x(t), t\in J_1$ is the constant solution of the system (1) so $x(t)$ is the constant solution on all interval J_0.

If $x(t_0), t_0\in J$ is constant solution of the system (1) so $g_i(t,x(t_0))=0$, $i\in\{1,2,\ldots,n\}$ for every $t\in J_0$.

Every integral curve of the constant solution $x\in D$ of the system (1) is parallel with axis o_t.

The solution $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))$, $t\in J_0$ of the system (1) called x_i – oscillation $i\in\{1,2,\ldots,n\}$ if $x_i(t)$ is oscillation function where the sequence $\{t_n\}_{n=1}^{\infty}$ exists thus $t_n\in J_0$, $t_n\to h$, $x_i(t_n).x_i(t_{n+1})<0$ for every $n\in N$.

The solution $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))$ called x_i – non-oscillation if exists $h_1<h$ thus $x_i(t)$ on the interval $\langle h_1,h)$(has on the interval $\langle t_0,h)$ of all limited number of any neutral points).

5 Literature

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The paper is support by grant 1/4004/2007.

Review: Doc. RNDr. Anna Tirpáková, CSc.

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