MATHEMATICAL MODEL AND DIFFERENTIAL EQUATION

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Abstract

The article introduces the result of the experimental dependencies of the maximum roughness of the cut surface. The maximum deviation is formulated by the differential equation.

Key words: Roughness of the cut surface, differential equation.

1 Introduction

In the investigation of the cut surface roughness it is marked the correlation inter multiple parameters that describe this operation. Multiple characteristics expressive the quality of the cut surface and it will be evaluate the effect of the tool. Rz is the maximum deviation of the cut surface roughness and Rz is function f of the tool feed and Rz is function r_ε of the tip radius. It is possible to expressive the correlative between the described parameters and also between the rate of the change respecting another independent variables.

2 Mathematical model of the relation $Rz = Rz(f, r_\varepsilon)$ by differential equation

The low cut speeds the rapid accruement of the roughness of the cut surface is expressed of the experimental dependencies of the maximum roughness of the cut surface Rz of the cut speed. Then the maximum deviation of the cut surface Rz is formulated by the equation

$$Rz^2 - 2Rzr_\varepsilon + \frac{f^2}{4} = 0 \tag{1}$$

In the following part there will be represented the maximum deviation of the cut surface roughness Rz as the function of the independent variable, that is the feed of the tool f then $Rz = Rz(f)$.

The differential equation with the derivative of the function $Rz = Rz(f)$ exists i.e. $\frac{dRz}{df} = Rz'$ and with analysis of the generalized solution of the differential equation it is possible to represent some connections of the maximum deviation of the cut surface roughness. Let the surface of the work be machined with the feed f with tool radius r_ε.

The equation (1) also represents the implicit function where other variable has therein before denotation.

The differential equation, where Rz is the unknown function is the equation (1), i.e. from the equation $F(f, Rz, r_\varepsilon) = 0$ (in (1) it is marked the right side $F(f, Rz, r_\varepsilon)$). The equation (1) is derivation of the variable f consequently from the equation $\frac{dF}{df} = 0$ is

$$2RzRz' - 2Rz'r_\varepsilon + \frac{f}{2} = 0 \tag{2}$$

where Rz' is the first derivative of the function Rz of the f .
From this equation results that the radius of the tool tip consequently parameter r_ε is expressed with the relation

$$r_\varepsilon = Rz + \frac{f}{4Rz'} \qquad (3)$$

where $Rz' \neq 0$. After substituting (3) in the relation (2) is

$$2fRz + 4Rz^2 Rz' - f^2 Rz' = 0 \qquad (4)$$

that is possible to represent

$$Rz'\left(4Rz^2 - f^2\right) - 2fRz = 0$$

The solution of the differential equation (4)

$$2fRz + 4Rz^2 Rz' - f^2 Rz' = 0$$

where Rz is the roughness of the cut surface, f is the feed and we determine the function Rz that Rz is the unknown quantity in differential equation (4) and is a function of the feed f , thus

$$Rz' = \frac{dRz}{df},$$

The solution is

$$Rz = \frac{c}{8} \pm \sqrt{\frac{c^2}{64} - \frac{f^2}{4}},$$

that system of relations expressed with the solution of the differential equation (4).
If we determine the initial conditions of the process for this solution where $\frac{c}{8} = r_\varepsilon$ is

$Rz = r_\varepsilon \pm \sqrt{r_\varepsilon^2 - \frac{f^2}{4}}$ and $r_\varepsilon^2 - \frac{f^2}{4} \geq 0$.

For the maximum deviate of the cut surface Rz we use the relation, $Rz = r_\varepsilon - \sqrt{r_\varepsilon^2 - \frac{f^2}{4}}$
where r_ε is the radius of tip tool and f is the feed of the tool.

4 Conclusion

The solution of the differential equation (4) where Rz is the roughness of the cut surface, f is the feed and we determine the function Rz that Rz is the unknown quantity in differential equation (4) and is a function of the feed f , thus $Rz' = \frac{dRz}{df}$. The differential equation is mathematical model. The experimental dependencies of the

maximum roughness of the cut surface *Rz* of the cut speed. The low cut speeds cause the rapid accruement of the roughness of the cut surface. Whit the increase of the cut speed the roughness of the surface decreases rapidly. The roughness of the cut surface is the exponential function and the graphic dependency is the part of the hyperbola.
The dependency of the roughness of the cut surface of the depth of the cut is formulated on the fundamental of the experiments. The depth of the cut has no marked effect on the roughness of the cut surface. The experimental dependencies are formulated with the linear function. The experiments provide the fundamental for the illustration and the mathematical analysis of the cut surface. The experiments are mostly for the low-carbon steels. The work investigates the effect of the some characteristics of the cutting process and studies roughness of the cut surface. There are elaborated theoretical information.

5 Literature

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