GRAPHIC REPRESENTATION OF THE SOLUTION DIFFERENTIAL EQUATIONS FIRST ORDER

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Resume

In the paper are the graphic representations of the solutions some differential equations first order. Graphs are constructed by using DERIVE.

Key words: solution of the differential equation, graphic representation, differential equation.

1 Introduction

We motivate students by the graphic representations of the solutions differential equations. The graphs of the solutions are constructed by using Derive. The graphic representations are applicable in the experiments, statistical study and science research.

2 First Order Differential Equations

We solve the equation

$$t - \frac{1}{t} + xx' = 0 \qquad (1)$$

Integrating with respect to t we obtain

$$t^2 + x^2 = \ln c_1 t^2, \quad c_1 \in \mathrm{R}^+ .$$

The graphic representative of the solution is on the next figure.

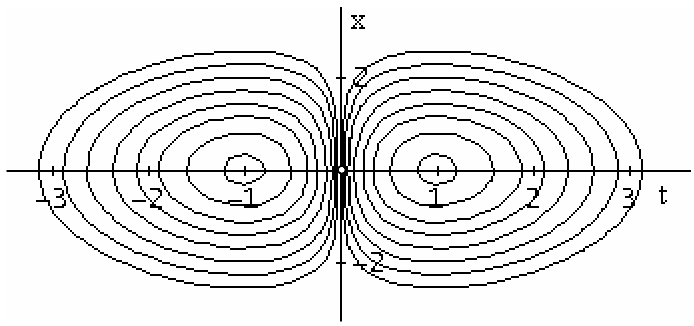

Fig1: *Graph of solutions of the equation* $t - \frac{1}{t} + xx' = 0$.

We solve the separable differential equation

$$(2t-1)(x^2+2x-3) + (2x+2)(t^2-t-2)x' = 0 \qquad (2)$$

Separating variables, we obtain

$$\frac{2t-1}{t^2-t-2} + \frac{2x+2}{x^2+2x-3}x' = 0 ,$$

where

$$\ln\left|t^2 - t + 2\right| + \ln\left|x^2 + 2x - 3\right| = c$$
$$\left(t^2 - t + 2\right)\left(x^2 + 2x - 3\right) = c \quad ,$$

where $c \in R\text{-}\{0\}$.

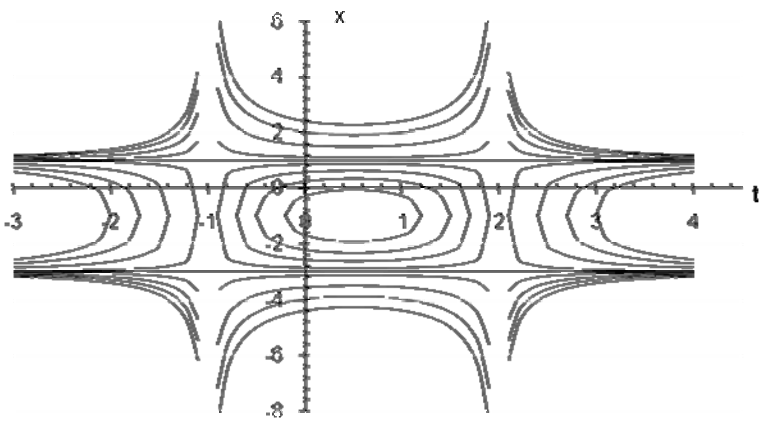

Fig 2: *Graf of solutions of the equation* $(2t-1)\left(x^2 + 2x - 3\right) + (2x+2)\left(t^2 - t - 2\right)x' = 0.$

If $t^2 + t + 3 = 0$, then $t \in \{-1,2\}$, and interval of validity is $R - \{-1,2\}$.
If $x^2 + 2x - 3 = 0$, then $x \in \{-3,1\}$, and the functions $x(t) = -3$, and $x(t) = 1$ are also the solution of the differential equation.

Solve the differential equation

$$t^2 x' + tx - x^2 = 0 \qquad (3)$$

Let $t \neq 0$, dividing of the equation $t^2 x' + tx - x^2 = 0$ by t^2,

$$x' + \frac{x}{t} - \left(\frac{x}{t}\right)^2 = 0$$

$$x = \frac{2tc}{c - t^2}, \quad c \in \mathrm{R} - \{0\}$$

we obtain too the solutions $x = 0$, or $x = 2t$.

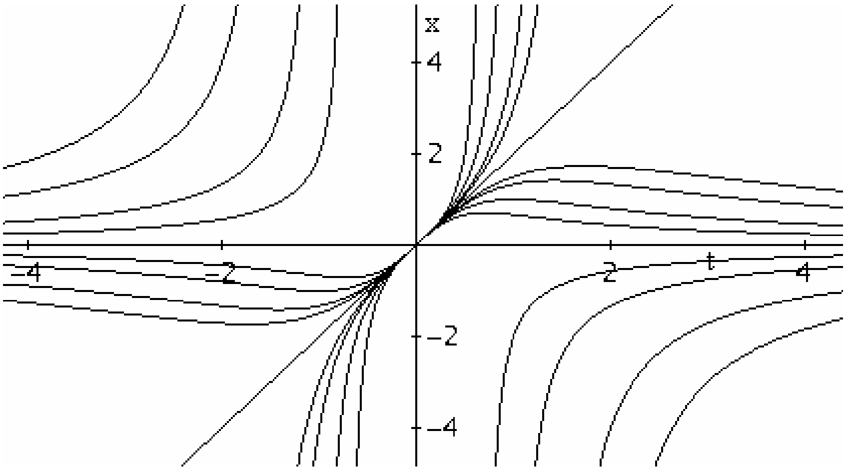

Fig. 3: *Graph of solutions* $t^2x' + tx - x^2 = 0$.

The differential equation

$$x' = (t - 4x + 3)^2 \tag{4}$$

Has the solutions

$$\frac{2t - 8x + 7}{-2t + 8x - 5} = ce^{2t}, \quad c \in \mathrm{R} - \{0\}$$

And other solutions

$$x = \frac{2t+5}{8}, \text{ or } x = \frac{2t+7}{8}.$$

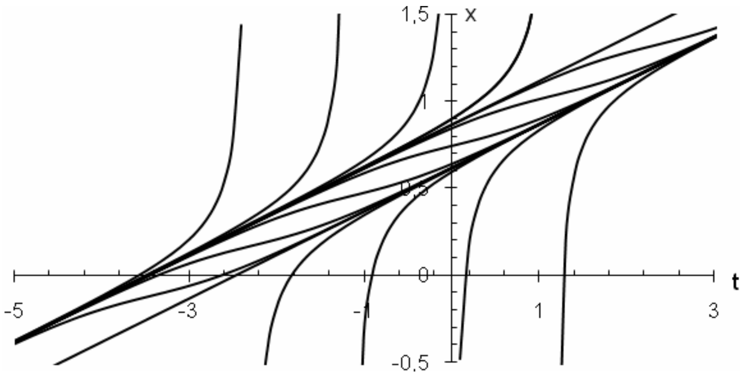

Fig. 4: *Graph of solution* $x' = (t - 4x + 3)^2$

3 Cauchy's initial value problem

If we solve the Cauchy's initial value problem

$$\frac{3e^t}{1-e^t}+\frac{\sec^2 x}{\operatorname{tg} x}x'=0\text{, and } x(\ln 2)=\frac{\pi}{4}. \qquad (5)$$

We first solve the differential equation

$$\frac{3e^t}{1-e^t}+\frac{\sec^2 x}{\operatorname{tg} x}x'=0$$

And the solution of the equation (5) are

$$-3\ln\left|1-e^t\right|+\ln\left|\operatorname{tg} x\right|=c,\ c\in R$$

The initial condition $x(\ln 2)=\frac{\pi}{4}$ requires that $x=\frac{\pi}{4}$ when $t=\ln 2$. Substituting this value in $-3\ln\left|1-e^t\right|+\ln\left|\operatorname{tg} x\right|=c$, we obtain $c=0$. Thus, the solution of the Cauchy's initial value problem is $-3\ln\left|1-e^t\right|+\ln\left|\operatorname{tg} x\right|=0$, or equivalently $\left|\operatorname{tg} x\right|=\left|1-e^t\right|^3$

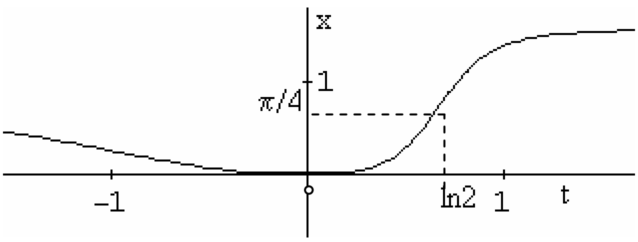

Fig 5: *Graf of solution* $\frac{3e^t}{1-e^t}+\frac{\sec^2 x}{\operatorname{tg} x}x'=0$*, and* $x(\ln 2)=\frac{\pi}{4}$.

4 Conclusions

In this text we have mathematical models of populations growth. Predicting the population at various times from this solution requires the evaluation of function at specific values of the variables. A more fundamental problem is determining how calculators compute these numbers or how a table is constructed.

5 References

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