A NOTELET TO MANIFOLD VIEWPOINT ON THE TORUS

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Abstract

There are various fields of communication where a word torus occurs in courses of technical university study. In this note we give a few areas of appearing such a 3-dimensional 2-parametric geometric object, named also toroid. The modest aim of our consideration is to show links joining the torus mathematics with diverse applications of scientific fields, especially with robot motion planning, the area important for technological university students, majoring in automation.

Key words: torus**,** toroid, anuloid, automation, robot motion, WinPlot.

1 Introduction

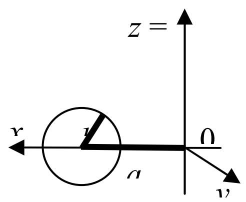
Likewise some acronyms have many meanings, similarly there are geometrical objects, which one can perceive and use from various viewpoints. The HP can serve as the example of multi-meaning acronym (a few of intents are given in appendix). The torus (named also tori, anuloid, tyre, surface of a doughnut, donut, wreath, etc.) is interesting 3d object with the usefulness from a very wide spectrum. Some points of view on the torus will follow in part 2.

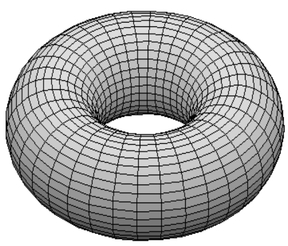
Physicist Ugo Amaldi in [1] brightly and appositely submits a cognition, that different fields of physics can all together bestead to the „beauty“ of our society by generating four streams: (i) basic knowledge, (ii) technologies, (iii) methods and (iv) people. And he also explains, that „the four output streams of any scientific endeavor are connected with four major activities of humankind: the construction of a unique scientific culture, and the growth of the world patrimony of wetware (the ability of brains to produce new ideas), software and hardware technologies“. The authors hope that presented divers stand-points on the toroid can – at least a little – contribute to the consciousness of these continuities.

2 Standpoints

Constructional Geometry

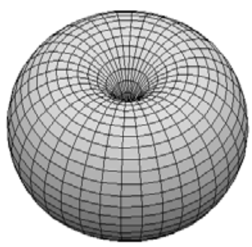
An anuloid belongs to the surfaces of revolution. A circle is the governing curve, placed in the rotational axis plane. Let's r denotes circle radius, a the distance from the circle center to the axis. Depending on r-a relation there are some possibilities of surface shapes and their designations correspondingly. Anuloid is one (the first) element of this set, shown on fig. 1.





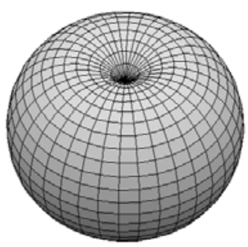
$a > r > 0$ (anuloid)

Fig. 1a



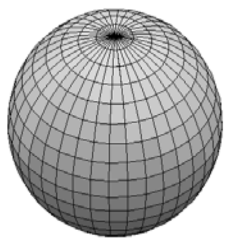
$a = r \neq 0$ (axoid)

Fig. 1b



$0 < a < r$ (melanoid)

Fig. 1c



$0 = a$ (sphere)

Fig. 1d

Analytic Geometry/Linear Algebra/Computer Graphics

An anuloid is the representative of the fourth order surfaces. Useful geometrical information (particularly for the students) in connection with the biquadratical surfaces, is the quantity of the intersections of any straight line and the surface: maximally four. Two of them are shown on fig. 2. For the purposes of anuloid PC portraying the parametric equations are more convenient than algebraic representation.

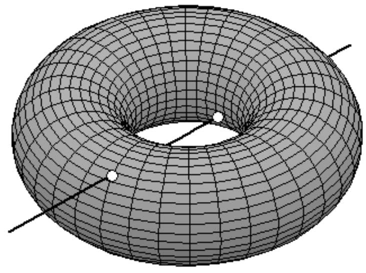


Fig. 2

Parametric equations of a directing circle are: $x(t) = r\cos t + a$, $y(t) = 0$, $z(t) = r\sin t$, $t \in \langle 0, 2\pi)$, and those of an anuloid: $x(t, u) = (r\cos t + a)\cos u$, $y(t, u) = (r\cos t + a)\sin u$, $z(t, u) =$ $= r\sin t$, $t \in \langle 0, 2\pi)$ $u \in \langle 0, 2\pi)$.

Deterministic Chaos Theory /Nonlinear Dynamics

Let's imagine IQ test. If one question is: "What belongs to the most powerful inventions of modern science?" One of the possible correct answers has to be the phase space [2], [3]. A strange attractor is the efficient visual icon of phase space. A toroid is an attractor, too – for the special motion type [4], [5].

Superstring Theory/High Energy Physics/ Quantum Geometry

There is a close relation between (1) the topological classification of string-theory diagrams and (2) the classification of the topology of 2-parametric surfaces [1], [7]. The sequence of Feynman diagrams is equivalent to a sequence of tori with increasing numbers of holes. The first three series members are figured on the picture below with the help of freely available open-source program *WinPlot* [6], [8]. (The third element, double-tori (fig. 3c), is just roughly shown; smoothness of transient part of this object is absented. This 2-anuloid is created by reciprocically moved two toruses.)

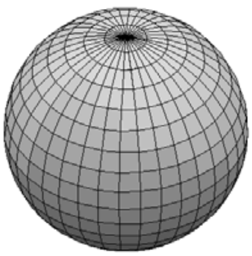


Fig. 3a: Sphere

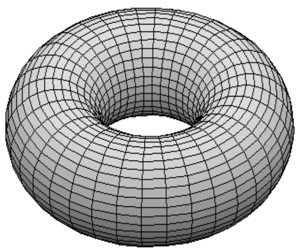


Fig. 3b: Torus

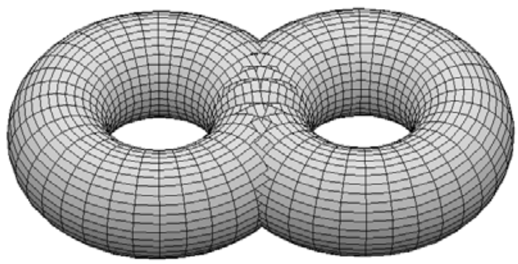


Fig. 3c: Double torus

Automation/ Robot Motion Planning

Overmastering the basic problems on 2-dimensional (Cartesian and planar) and 3-dimensional (Cartesian, cylindrical and spherical) coordinate system in a framework of calculus and analytical geometry course, technological universities students – especially those, majoring in automation branches – are ready to familiarize themselves with traditional set of spaces, used in a robot motion planning: work space, configuration space and a topological space [9], [10]. Fig. 4a shows 2-link robot arm in a configuration, characterized by a pair of angles; the angle α has its vertex in the first joint, and the angle β in the second one. Both angles are measured from the positive x-axis direction. Any position of the robot arm in the workspace (0; x, y) corresponds to a point in the configuration space (0; α, β) figured in 4b. Topological space (fig. 4c) of a tori-shape illustrates robot arm configuration in a next way, belonging in mathematics to the manner of high elegancy ☺.

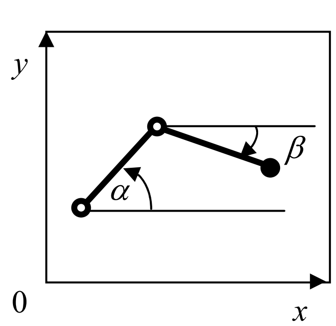


Fig. 4a: *Work space.*

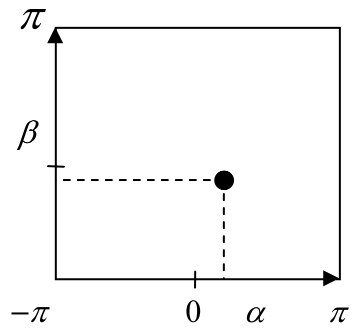


Fig. 4b: *Configuration space.*

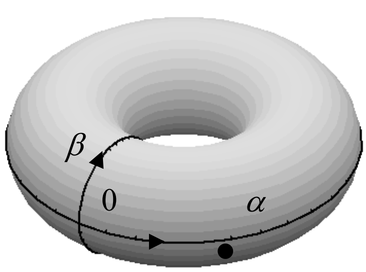
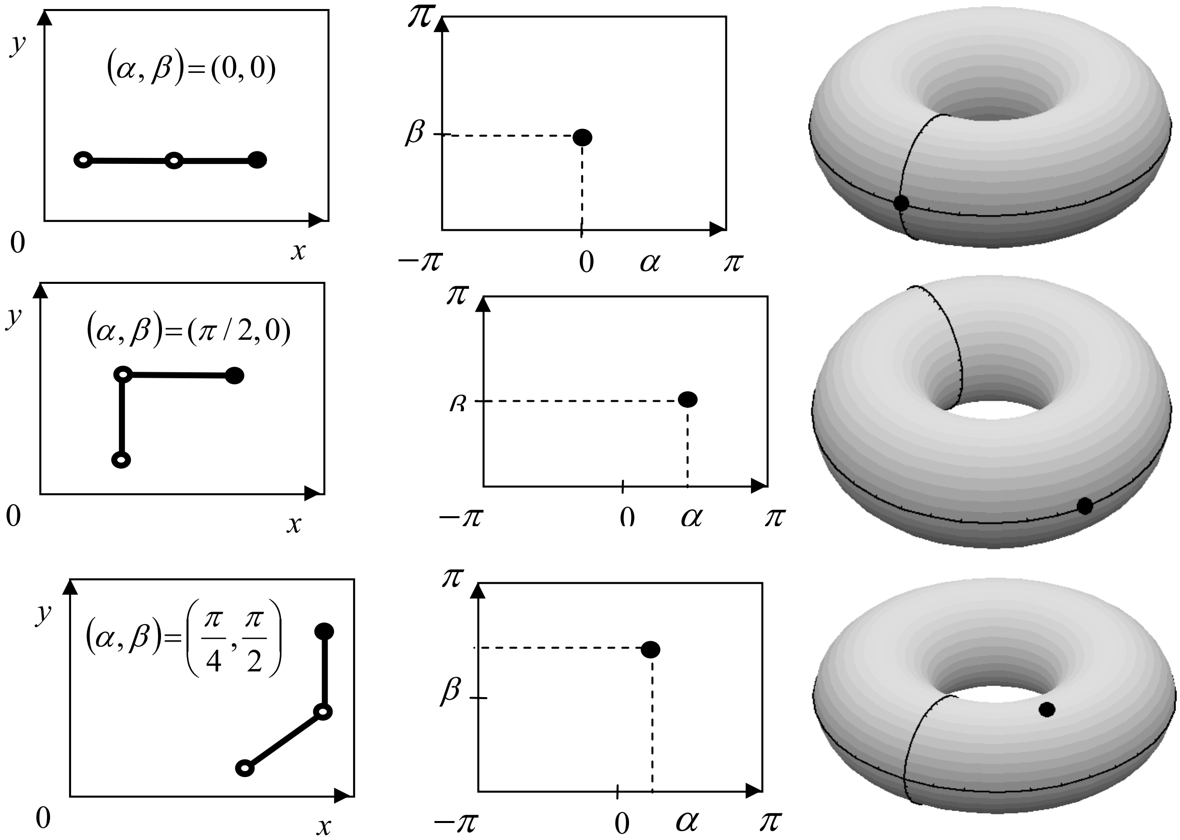


Fig. 4c: *Topological space.*

Some of the simplest (α, β) pairs are figured below; they probably can serve like little inspiration/appendix to elementary mathematical course chapter on coordinate systems.



3 Conclusion

The paper has presented some different viewpoints on a geometrical object called torus. The diversity of standpoints from various mathematical/physical/application fields is unifying element of their list. A bit more attention has been paid to an automation/robot motion planning branch.

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Appendix

Following collection of HP acronym diverse meanings was „constituted“ by saddle surface **h**yperbolical **p**araboloid and supplemented thanks to students creations: **h**orse **p**ower, **H**ewlett-**P**ackard, **H**arry **P**otter, **H**abera **P**avol, **h**nusná **p**rednáška ☹, ...

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