LINEAR DIFFERENTIAL SYSTEM OF THE FIRST ORDER AND THE NEW METHOD FOR DETERMINED THE FUNDAMENTAL MATRIX

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Abstract

The method of the auxiliary numbers of the theory linear differential systems of the first - order with the constant coefficients is apply on the linear differential system of the first order with using the sequences matrix.

Key words: differential system, sequence matrix.

1 Introduction

We consider about linear differential system

$$x_i'(t) = (a(t)A_n + b(t)E_n)x_i(t), \quad i = 1,2,...,n, \quad n \geq 4, \tag{1}$$

where $a(t), b(t) \in C_0$, $a(t) \neq 0$ for all $t \in J$, C_0 is the space of the all continuous real functions of the real variable define on the interval J.
For elements a_{ij} of the matrix A_n is valid: $a_{ij} = 0$, if $i = j$ or $i + j = n + 1$, for $i = 1, 2, \ldots, n$ and for every $i = 2,3, \ldots, n-1$, $j = 2,3, \ldots, n-1$, $a_{i1} = -1$, for every $i \neq 1$, $i = 2,3, \ldots, n-1$, $a_{1j} = 1$ and $a_{nj} = 1$ for every $j \neq 1, j \neq n$, $a_{in} = 1$, if $i \neq 1$, $n \geq 4$. E_n is identity matrix.
We denote

$$D_n(t) = a(t)A_n + (b(t) - \lambda(t))E_n. \tag{2}$$

Definition 1 The solution $\lambda(t), t \in J$ of the auxiliary equation $|D_n(t)| = 0$ is called „auxiliary function“ of the matrix system (1)[1].

2 Construction fundamental matrixes

Theorem 1 The vector function $X(t) = (\xi_1(t), \xi_2(t), \ldots, \xi_n(t))C, t \in J$, when $C = (c_1, c_2, \ldots, c_n)^T, c_i \in R, i = 1,2,\ldots,n$ is the vector constant and the vector function $\xi_1(t), \xi_2(t), \ldots, \xi_n(t)$ are columns of the fundamental matrix .
$U(t) = q_1(t)P_0 + q_2(t)P_1 + q_3(t)P_2 + \ldots + q_m(t)P_{m-1}$, $m \leq n-1, n \geq 4, m,n \in N$ of the system (1) is the general solution of the system (1) [2],[3],[4].
Lemma 2 Let $m \leq n-1, n \geq 4, m,n \in N$, that $A_n^m = 0$,when A_n is matrix of the system (1)[5].
Proof We constitute the sequence with the matrixes

$$P_0 = E_n, \; P_1 = \frac{B_n(t) - b(t)E_n}{a(t)} = A_n, \; P_2 = A_n^2, P_3 = A_n^3, \ldots, P_{n-2} = A_n^{n-2}, \; P_{n-1} = A_n^{n-1} = O$$

(neutral matrix), we determine $P_{n-1} = P_m$, then $A_n^m = O$, $m \leq n-1, n \geq 4, m,n \in N$.
It follows of the Lemma 2 for own functions $\lambda_1(t) = \lambda_2(t) = ... = \lambda_n(t) = b(t)$. We constitute the sequence of the differential equations

$$q_i'(t) = b(t)q_1(t), \qquad q_1(t_0) = 1,$$

$$q_2'(t) = b(t)q_2(t) + a(t)q_1(t), \qquad q_2(t_0) = 0,$$

$$q_3'(t) = b(t)q_3(t) + a(t)q_2(t), \qquad q_3(t_0) = 0, \tag{5}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$q_m'(t) = b(t)q_m(t) + a(t)q_{m-1}(t), \qquad q_m(t_0) = 0,$$

$m \le n-1,\ n \ge 4,\ m,n \in N$

The functions (6) are solutions of the system (5)

$$q_1(t) = \exp\int_{t_0}^{t} b(s)ds,$$

$$q_2(t) = \int_{t_0}^{t} a(s_1)ds_1 \exp\int_{t_0}^{t} b(s)ds \tag{6}$$

$$q_3(t) = \left(\int_{t_0}^{t}\left(a(s_1)\int_{t_0}^{s_1} a(s_2)ds_2\right)ds_1\right)\exp\int_{t_0}^{t} b(s)ds$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$q_m(t) = \int_{t_0}^{t}\int_{t_0}^{s_{m-1}}\ldots\int_{t_0}^{s_2}\prod_{i=1}^{m-1} a(s_i)ds_1 ds_2 \ldots. ds_{m-1} \exp\int_{t_0}^{t} b(s)ds,$$

We show, that the fundamental matrix of the system (1) is in the form

$$U(t) = q_1(t)P_0 + q_2(t)P_1 + \ldots + q_m(t)P_{m-1} \tag{7}$$

or

$$U(t) = \sum_{i=1}^{m} q_i(t)P_{i-1}\ .$$

We denote with differentiate equations (7)

$$U'(t) = \sum_{i=1}^{m} q_i'(t)P_{i-1} = b(t)q_1(t)P_0 + \sum_{i=2}^{m} b(t)q_i(t) + a(t)q_{i-1}(t)P_{i-1} = b(t)\sum_{i=1}^{m} q_i(t)P_{i-1} +$$

$a(t)\sum_{i=1}^{m} q_i(t)P_i = b(t)U(t) + a(t)\sum_{i=1}^{m} q_i(t)P_i$, according to Lemma 2, $P_m = P_{m-1}P_1 = A_n^m = 0$.

Then $U'(t) = b(t)U(t) + a(t)P_1\sum_{i=1}^{m} q_i(t)P_{i-1} = (b(t)P_0 + a(t)P_1)U(t)$, hence $U(t)$ is the fundamentally matrix of the system (1), thus columns of the matrix $U(t)$ are the linearly independent solutions of the differential system (1). The general solution of the system (1) is in the form

$X(t) = U(t).C$,

where $C = (c_1, c_2, \ldots, c_n)^T, c_i \in R,\ i = 1,2,\ldots,n$ is the constant vector.

3 Conclusion

The text presents the solution of the problem of the process which is expressed by the mathematical model, i.e. by a system of differential equations and the optimal criterion is represented by the function. Differential equations systems are mathematically studied from mostly concerned with their solutions, functions that make the equation hold true. Only the simplest differential equations admit solutions given by explicit formulas [6], [7]. Many properties of solutions of a given differential equation may be determined without finding their exact form. If a self-contained formula for the solution is not available, the solution may be numerically approximated using computers**.**

4 Literature

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