A NOTELET TO CLICHÉ-CLAIM ON MATHEMATICAL TRANSPARENCY

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Abstract

The article presents a simple classroom example concerning the obligatory question how to make explaining the conception of the definite integral more transparent for technical university students. The explanation takes into account such mathematical fields as mathematical analysis, analytic geometry, linear algebra, as well as computer geometry.

Key words: mathematical analysis, analytic geometry, linear algebra, computer geometry, WinPlot.

1 Introduction

In this paper we prefer the task to illuminate for students, future graduate technological engineers, the mathematical conception of definite integral in a more general form than by using exclusively tools of mathematical analysis, especially (i) computing the volume (with parallel cross sections of the revolving solid; [1], [2]). We complement this familiar way by (ii) analytical expressions of some 2d and 3d objects via parametric equations, this means within the framework of analytical geometry; next by (iii) linear mappings and their composition, known in linear algebra; and finally by (iv) visualization of geometrical objects given by (ii) and (iii), i. e. with the help of computer geometry [3]. We also add to (iv) *WinPlot* [4], a freely available open-source program that has been used to visualize planar and spatial elements.

Steps mentioned above are used (i) to compute the volume of the bucket ☺, (ii) to analytically express its elements (i. e. a bottom, a jacket, a handle), (iii) to transform the handle from a vertical position to the down one and (iv) using expressions from a step (ii) to figure the bucket via *WinPlot*.

2 Relations

(i) The volume of the bucket is the volume of a solid of revolution formed by revolving trapezium, as it is shown on fig. 1a,

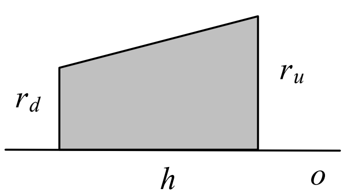

Fig. 1a

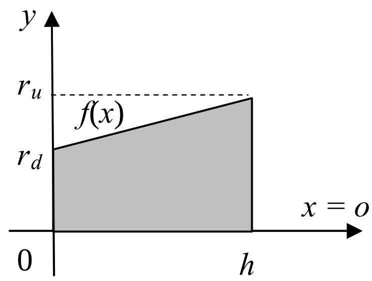

Fig. 1b

where o – rotational axis, h – height of the bucket, r_d, r_u – down and upper radius correspondingly. Placing trapezium in 2d Cartesian coordinate system (fig. 1b) and putting

$f(x) = \frac{r_u - r_d}{h} x + r_d$, volume of the bucket – by using the known formula – is $V = \pi \int_0^h \left(\frac{r_u - r_d}{h} x + r_d \right)^2 dx.$

(ii) To visualize a bucket via PC the parametric equations are the most convenient ones. Assuming the position of the bucket in 3d orthogonal right-handed coordinate system (*O*; *x*, *y*, *z*) with *z*-axis as the one of revolution, the bucket bottom as a disk in *xy*-plane, the bucket jacket as a conical surface part with the governing straight line segment in *xz*-plane, and the bucket handle in upper/vertical position as a semicircle in *xz*-plane, we have:

the bottom: $x(t, u) = t\cos u$, $y(t, u) = t\sin u$, $z(t, u) = 0$, $t \in \langle 0, r_d \rangle$ $u \in \langle 0, 2\pi)$, fig. 2a,

the straight line segment: $x(t) = t$, $y(t) = 0$, $z(t) = (t - r_d)\frac{h}{r_u - r_d}$, $t \in \langle r_d, r_u \rangle$, fig. 2b,

the jacket: $x(t,u) = t\cos u$, $y(t,u) = t\sin u$, $z(t,u) = (t - r_d)\frac{h}{r_u - r_d}$, $t \in \langle r_d, r_u \rangle$, $u \in \langle 0, 2\pi)$, fig. 2c,

the handle: $x(t) = r_u\cos t$, $y(t) = 0$, $z(t) = r_u\sin t + h$, $t \in \langle 0, \pi \rangle$, fig. 2d.

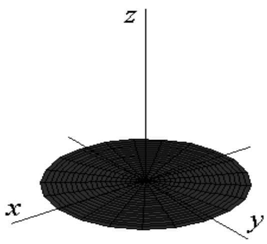

Fig. 2a

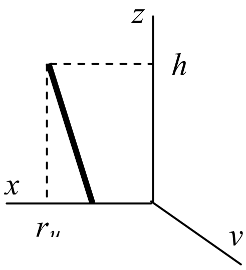

Fig. 2b

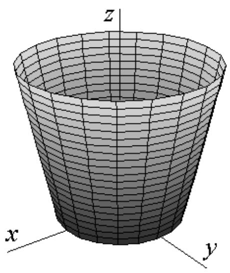

Fig. 2c

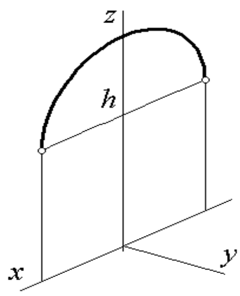

Fig. 2d

(iii) To figure handle in a down position, composite mapping has to be realized. It consists of three elementary linear conforming mappings. Parametric equations of the handle are given in each of mapped positions:

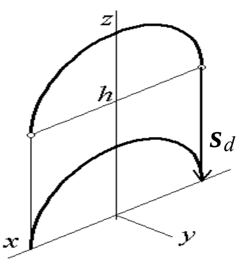

Fig. 3a

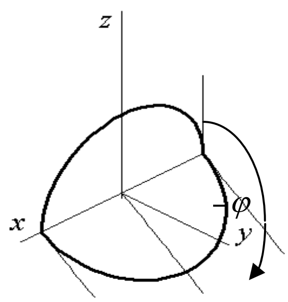

Fig. 3b

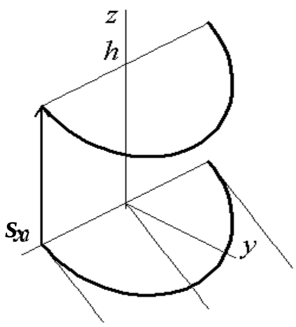

Fig. 3c

a) translation $\boldsymbol{T}_{(0,\,0,\,-h)}$ with translational vector $\boldsymbol{s_d} = (0,0,-h)$, $h > 0$, fig. 3a,

$x(t) = r_u\cos t,\ y(t) = 0,\ z(t) = r_u\sin t,\ t \in \langle 0, \pi \rangle$,

b) rotation $\boldsymbol{R}_{(x,\,-\varphi)}$ by an angle $-\varphi$ about rotational axis $o = x$, $\varphi > 0$, fig. 3b,

$x(t) = r_u\cos t,\ y(t) = -r_u\sin t\cos\alpha,\ z(t) = r_u\sin t\sin\alpha,\ t \in \langle 0, \pi \rangle$,

c) translation $\boldsymbol{T}_{(0,\,0,\,h)}$ with translational vector $\boldsymbol{s_u} = (0,0,\ h)$, $h > 0$, fig. 3c,

$x(t) = r_u\cos t,\ y(t) = -r_u\sin t\cos\alpha,\ z(t) = r_u\sin t\sin\alpha + h,\ t \in \langle 0, \pi \rangle$.

(Depending on graphical software possibilities, transformational matrices/equations can be used to figure the actual object in transformed (translated or rotated) position.)

The figures 4a, b illustrate an angle φ sense and derivation.

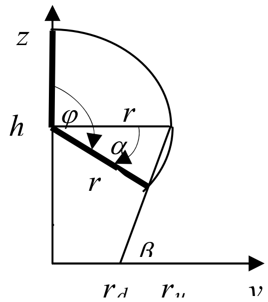

Fig. 4a

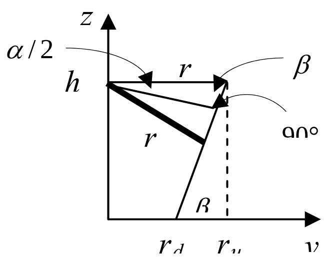

Fig. 4b

$$\alpha = 2.\left(\pi - \frac{\pi}{2} - \beta\right)$$

$$\beta = arctg\,\frac{h}{r_u - r_d}$$

3 Numerical Example

Let`s assume an existence of an ordinary household bucket with the volume of about 10-15 l, h = 25 cm, r_d = 10 cm, r_u = 15 cm. $\varphi = 90° + \alpha$, $\alpha = \pi - 2\mathrm{arctg}\,\dfrac{h}{r_u - r_d} \approx 22.5°$.

Computing the bucket volume approximately, we have $V = \pi\displaystyle\int_0^{25}\left(\frac{x}{5} + 10\right)^2 dx =$

$$= \pi\left[\frac{x^3}{3.25} + 4.\frac{x^2}{2} + 100x\right]_0^{25} = \pi\left(\frac{25^2}{3} + 2.25^2 + 4.25.25\right) = \pi.25^2.\frac{19}{3} \approx 625.20 = 12500\ (\mathrm{cm}^3),$$

i. e. 12.5 l.

The problem of two surfaces intersection line figuring appears in showing the handle down position (fig. 5b), where the aim was to show the handle points as the subset of the jacket points. (To make happy unmathematically oriented observer ☺, easy alternative exists to figure the whole, continuous handle, without gaps: to use a bit longer radius, e. i. $r + \Delta r$ – see fig. 5c, or more illustrative – fig. 5d).

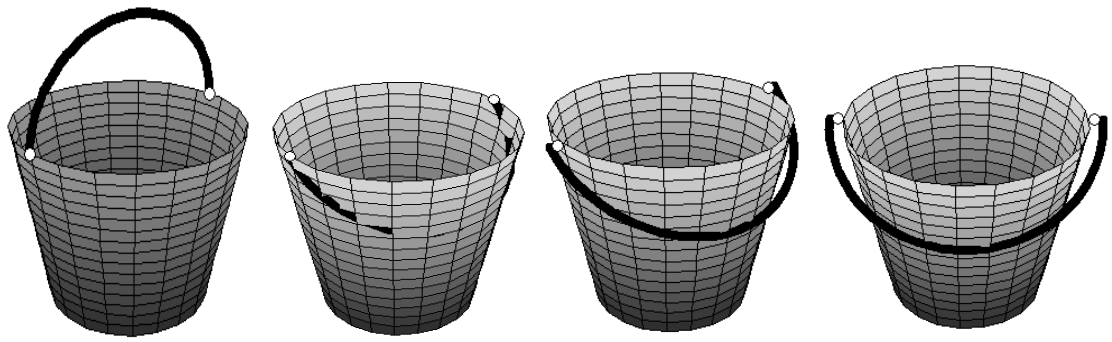

Fig. 5a Fig. 5b Fig. 5c Fig. 5d

4 Conclusion

An elementary classroom problem determined to better illumination of definite integral notion was presented here. To extend the knowledge of this notion and emphasize the empirical impact on understanding possible physical processes, we have combined four mathematical branches including the modern exposition tool – computer geometry. There are next fields omitted though useful and interesting, too. E. g. constructional geometry: developing drawing skills by sketching – with respect to an affine attributes of conic sections – is almost the necessary skill for students, without respect to the graphical program applications level. Or hydromechanics – from the contunuum mechanics view point: quite interesting problem might be identifying the surface shape of rotating liquid in the conical vessel (about the axis of revolution) if compared with the surface shape in cylindrical container.

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