A NOTELET TO VISUALIZED INTEGRAL CURVES CHOICE

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Abstract

An article presents a small illustration on a differential geometry and combinatory symbiosys in a choice of visualized integral curves. Visualization is performed via *Maxima*, a freely available open-source program.

Key words: differential equation, integral curve, combinatorics, differential geometry, computer algebra system *Maxima.*

DROBNÁ POZNÁMKA K VÝBERU ZOBRAZOVANÝCH INTEGRÁLNYCH KRIVIEK

Resumé

Článok predkladá drobnú ilustráciu symbiózy diferenciálnej geometrie a kombinatori-ky pri výbere zobrazovaných integrálnych kriviek. Vizualizácia je vykonaná pomocou voľne dostupného open-source programu *Maxima*.

Kľúčové slová: diferenciálna rovnica, integrálna krivka, kombinatorika, diferenciálna geometria, systém počítačovej algebry *Maxima.*

Introduction

An introductory university mathematics course for engineering students consists of various math branches basic knowledge, including subject differential equations (DEs). The solution of DE, also (geometrically) presented with synonymic indication as an integral curve, contains a set of integrating constants (parameters). Their quantity corresponds to the DE order. Their values result from (given) initial conditions (e. g. [1], [2], [3], [4]).

In this paper the values of integral parameters don`t result from initial conditions completely. Here a) just one (of more) initial condition is prescribed – with respect to a differential geometry point of view, then b) integrating parameters values are chosen – on the combinatory approach base, and finally c) remaining initial conditions values are computed.

Visualization of integral curves is realised with the uncommercial system *Maxima* [5]. Zero cost and unneeded licensing agreement permit students to experiment with software away from the computer laboratories at the university [6]. Although open-source computer algebra system *Maxima* is not as full-featured and powerful as e. g. Maple and Mathematica, it is excellently suited for a graphs visualizing. Students response to the introduction of this system has been very positive.

Standpoints

Combinatorics: The third-order (linear) DE is given: $y''' = 1$. General solution is of the form $y(x) = \frac{x^3}{6} + c_1 x^2 + c_2 x + c_3$; c_1, c_2, c_3 are arbitrary real constants. Let`s their values are the variations of two numbers 0 and 1 taken (with repetition) three at a time, e. i. the number of triplets is $\overline{V}_3(2) = 2^3 = 8$ and their values are $(c_1, c_2, c_3) \in$ {(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 0), (1, 0, 1), (0, 1, 1), (1, 1, 1)}. The fig. 1. presents integral curves graphs for these alternatives.

Differential geometry and combinatorics: The third-order linear DE with constant coefficients $y''' - y'' - 2y' = 0$ is given. Characteristic equation roots are 0, – 1, 2, and the general solution $y(x)$ has the form

$y(x) = c_i y_i = c_1 + c_2 e^{-x} + c_3 e^{2x}$, $c_i \in R$, $i = 1, 2, 3$.

A) Let`s fix ***x*** and ***y*(*x*)**: e. g. $y(0) = 2$, i. e. the requirement of all ICs have to contain **the same point** (0, 2), is set.
$y(x) = c_1 + c_2 e^{-x} + c_3 e^{2x}$, $y(0) = c_1 + c_2 + c_3$, $c_1 + c_2 + c_3 = 2$;
$(c_1, c_2, c_3) \in$ {e. g. (0, 0, 2), (0, 1, 1), (0, – 1, 3), (3, 1, – 2), (1, 2, – 1)}.
There are integral curves graphs for these alternatives shown on fig. 2a) as well as tangents to these curves in their common point; tangents are of a **different slopes**.

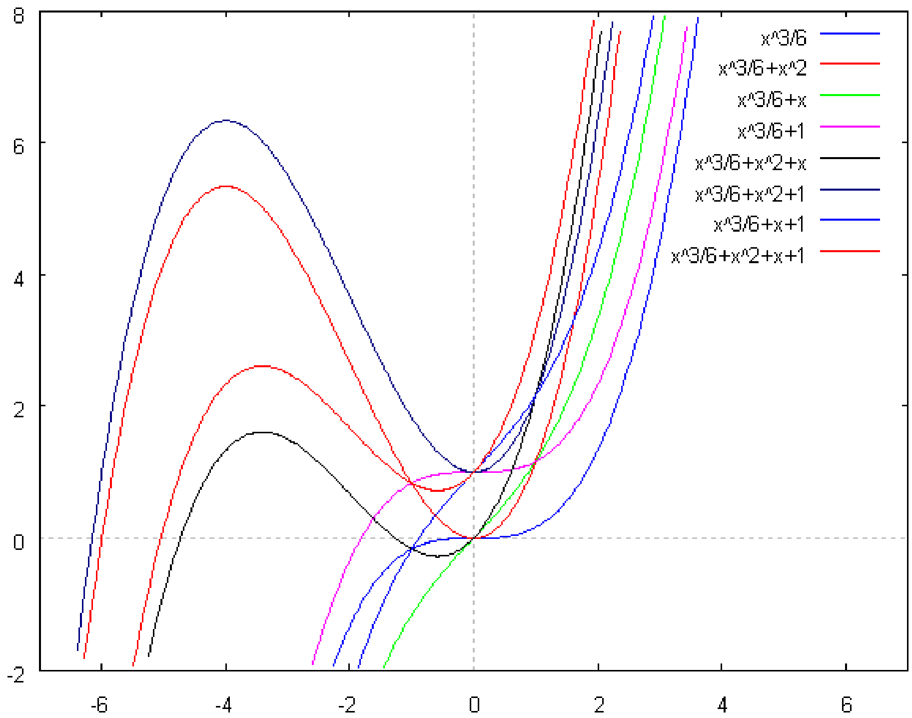

Fig. 1. Integral curves with combinatorily chosen integrative parameters c_1, c_2, c_3

B) Let`s fix ***x*** and ***y*´(*x*)**: e. g. $y'(0) = 1$, i. e. the requirement of all ICs have their **tangent** to the point with the *x*-coordinate $x = 0$ of **the same slope**, is set.
$y'(x) = - c_2 e^{-x} + 2c_3 e^{2x}$, $y'(0) = - c_2 + 2c_3$, $- c_2 + 2c_3 = 1$;
$(c_1, c_2, c_3) \in$ {e. g. (0, 1, 1), (0, 0, 1/2), (0, 2, 3/2), (0, – 1, 0), (0, – 1/2, 1/4)},
Fig. 3. presents corresonding integral curves; they have **no common intersection** unlike on fig. 2 but the tangents to their points (0, *y*(0)) have **an identical rate**.

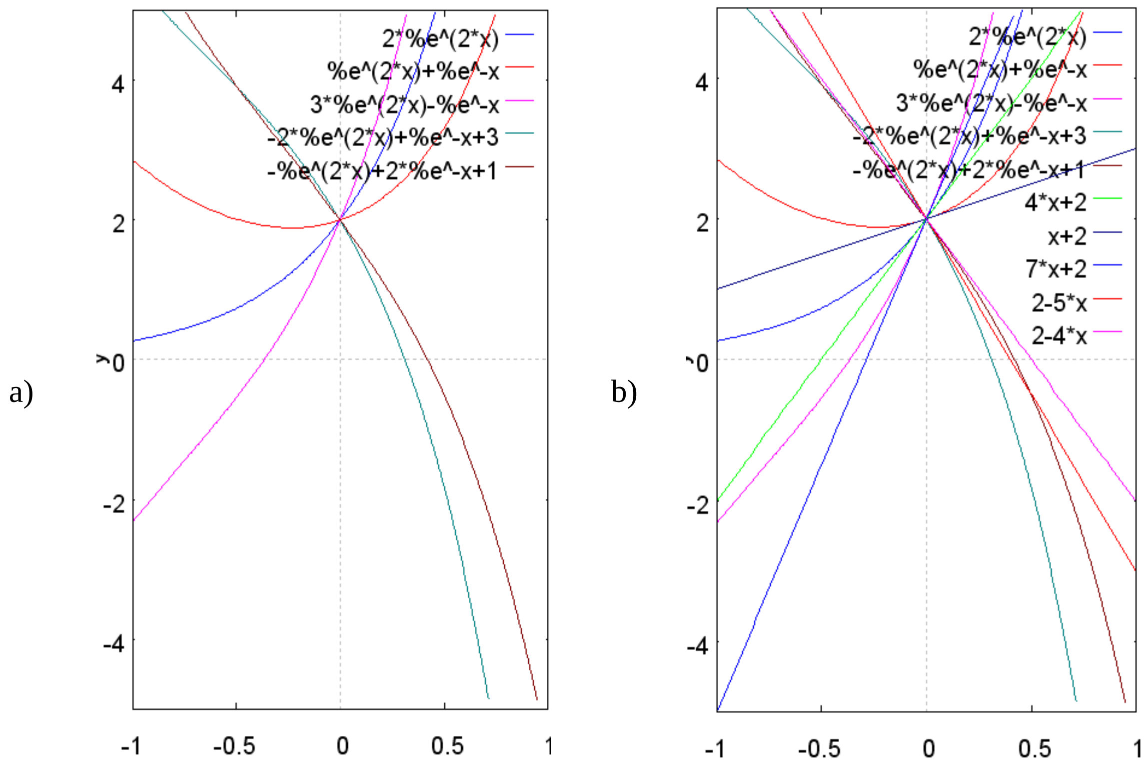

Fig. 2. a) Integral curves contain **the same point** (0, 2); b) their tangents to this point are of a **different rates**

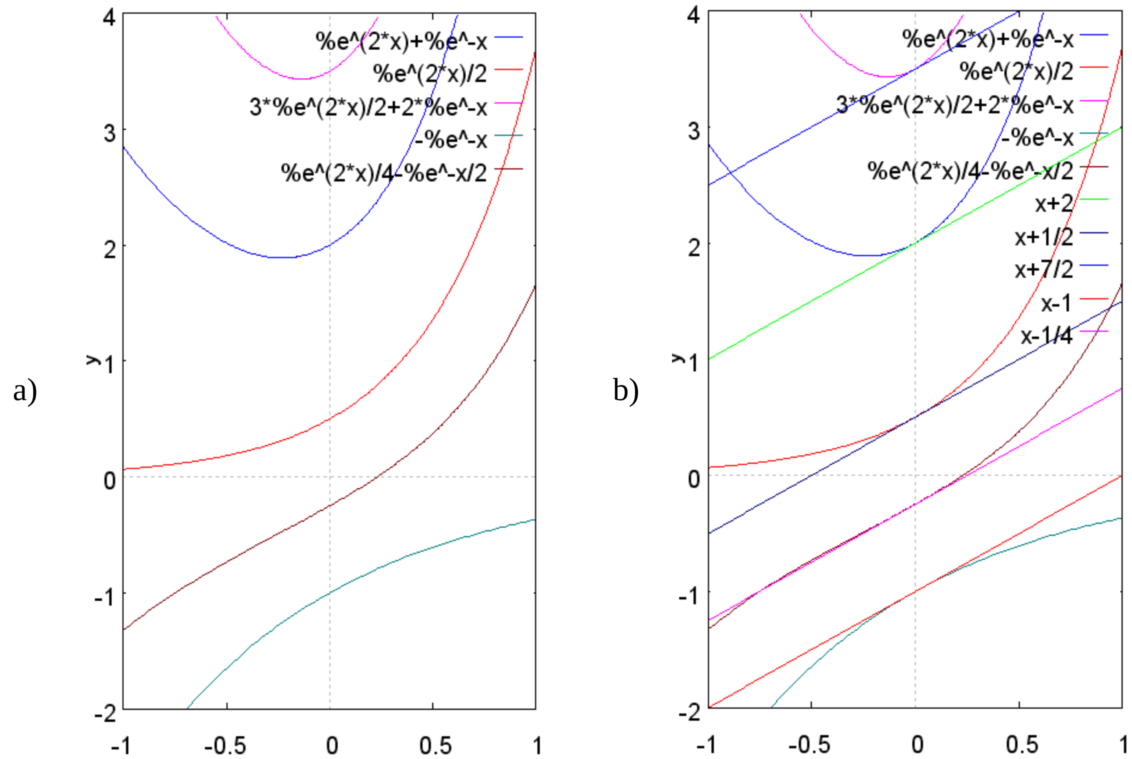

Fig.3. a) Integral curves haven\`t **any common intersection** unlike on fig. 2; b) their tangents to their points with *x*-coordinate $x = 0$, i. e. to $(0, y(0))$, have **the same rate**.

4 Conclusion

The paper has presented two standpoints (combinatorics and differential geometry one) on a choice in visualization of DE solutions. Open-source computer algebra system *Maxima* has served as a satisfactorily powerful equipment to visualize them.

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Appendix

Maxima input instructions to generate graphs shown on fig. 2

a) plot2d([2*%e^(2*x),%e^(-x)+%e^(2*x),-%e^(-x)+3*%e^(2*x),3+%e^(-x)-2*%e^(2*x), 1+2*%e^(-x)-%e^(2*x)], [x,-1,1], [y,-5,5], [plot_format, gnuplot])$

b) plot2d([2*%e^(2*x),%e^(-x)+%e^(2*x),-%e^(-x)+3*%e^(2*x),3+%e^(-x)-2*%e^(2*x), 1+2*%e^(-x)-%e^(2*x),4*x+2,x+2,7*x+2,-5*x+2,-4*x+2], [x,-1,1], [y,-5,5], [plot_format, gnuplot])$

and on fig. 3.

a) plot2d([%e^(-x)+%e^(2*x),%e^(2*x)/2,2*%e^(-x)+3*%e^(2*x)/2,-%e^(-x), -%e^(-x)/2+%e^(2*x)/4], [x,-1,1], [y,-2,4], [plot_format, gnuplot])$

b) plot2d([%e^(-x)+%e^(2*x),%e^(2*x)/2,2*%e^(-x)+3*%e^(2*x)/2,-%e^(-x), -%e^(-x)/2+%e^(2*x)/4,x+2,x+1/2,x+7/2,x-1,x-1/4],[x,-1,1],[y,-2,4], [plot_format, gnuplot])$

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