TECHNICAL EDUCATION AND COMPUTING SUPPORT OF MATHEMATICS APPLICATIONS

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Abstract

The paper presents the possibilities of MS Excel and Matlab program equipment utilisation with mathematics applications to engineering subjects at technical universities.

Key words: technical education, mathematics applications, MS Excel, Matlab.

TECHNICKÉ VZDELÁVANIE A POČÍTAČOVÁ PODPORA APLIKÁCII MATEMATIKY

Resumé

V príspevku sú prezentované možnosti využitia MS Excelu a Matlabu pri aplikáciách matematiky v inžinierskych predmetoch na technických univerzitách.

Kľúčové slová: technické vzdelávanie, matematické aplikácie, MS Excel, Matlab.

Introduction

In order to make mathematics education at faculties of technical universities more effective, mathematics teachers give more effort to the implementation of computers to the teaching process. It is expected that in the course of university technical education with the assistance of mathematics teachers students will learn how to utilize the advantages of suitable program systems with the solution of mathematical applicable tasks in other engineering subjects. Based on the specific example, the paper will present the utilisation of the program system Matlab and MS Excel program with the applications of differential and non-linear equations in the engineering subject Elasticity and Strength.

1 PC Utilisation with Mathematics applications to Engineering Subjects

Within the subject *Elasticity and Strength* students‘ task is to evaluate a rod stability, i.e. to determine Euler's critical force F_k and a critical stress σ_k for different ways of rod's ends placing, which always leads them to the solution of a linear differential equation of the 2nd order with constant coefficients. When determining F_k and σ_k for the rod with a constant cross-section and the area *A*, whose one end is fixed and the other one is hinged, it is necessary to solve a differential equation:

$$\frac{d^2 w(x)}{dx^2} + \alpha^2 w(x) = \frac{M_A}{EJ}\left(1 - \frac{x}{l}\right) \tag{1}$$

which is satisfied by the bending curve $w(x)$. A general solution to the equation (1) will be found by the use of a characteristic equation and a method of indefinite coefficients (Vagaská, 2006) in the shape:

$$w(x) = C_1 \cos\alpha\, x + C_2 \sin\alpha x + \frac{M_A}{F_k} - \frac{M_A}{F_k l} x \tag{2}$$

whence

$$w'(x) = -\alpha\, C_1 \sin\alpha\, x + \alpha\, C_2 \cos\alpha\, x - \frac{M_A}{F_k l} \tag{3}$$

Utilising the boundary conditions $w(0) = w(l) = 0$ (for this rod there is a zero deflection at end points) and $w'(0) = 0$ (at the end where the rod is fixed there is a zero moving around) in (2) and (3) we will obtain a set of three equations with three unknowns C_1, C_2 a $\frac{M_A}{F_k}$. As we want a non-trivial solution, determinant of the mentioned set has to be zero (Vagaská, 2006).Based on this condition, by the help of Saruss' rule we will obtain the equation

$$tg\alpha l = \alpha l \text{ , resp. } tg\, x = x \qquad (4)$$

after a substitution introduction $\alpha l = x$. A professional literature related to *Elasticity and Strength* brings only a graphical solution to the equation (4) and the smallest positive root with the accuracy $\varepsilon = 10^{-3}$, i.e. $\alpha l = 4{,}493$ (Trebuňa, 2000).

After unknown α is found, for the smallest value of Euler's critical buckling force we obtain: $F_k = \alpha^2 EJ = \frac{(4{,}4934)^2}{l^2} EJ = \frac{\pi^2 EJ}{(0{,}699 l)^2} \doteq \frac{\pi^2 EJ}{(0{,}7 l)^2}$. Let us show a graphical and numerical solution to the non-linear equation(4) with the use of PC.

2 Graphical Solution to Non-Linear Equations with Matlab

Graphs points of intersection of the functions $f: y = tg\, x$ and $g: y = x$ are the solution to the equation (4). We will write these functions in the Matlab environment:

```
>> x=[2:0.02:4.6];
>> y=tan(x);
>> z=zeros(size(x));
>> y2=x;
>> plot(x,y,x,z,x,y2);
```

and after being proved, we obtain a graphical solution that can be seen in fig.1, whence $\alpha \cong 4{,}5$. In the first order determining the interval of a variable x the requirement of finding the smallest positive root of the given transcendent equation has been taken to consideration.

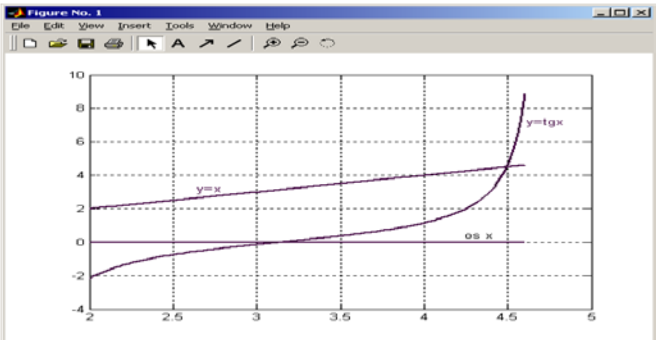


Fig.1 *Graphical solution to the non-linear equation $tg\, x = x$ in the Matlab environment*

3 MS Excel with Graphical and Numerical Solution to Non-Linear Equations

With the graphical solution to the equation $tg\, x = x$ (fig. 2), from the viewpoint of graphs designing simplicity, the utilisation of MS Excel seemed to be less suitable. The graph of the function $y = tg\, x$ had to be designed in parts considering the points of discontinuity.

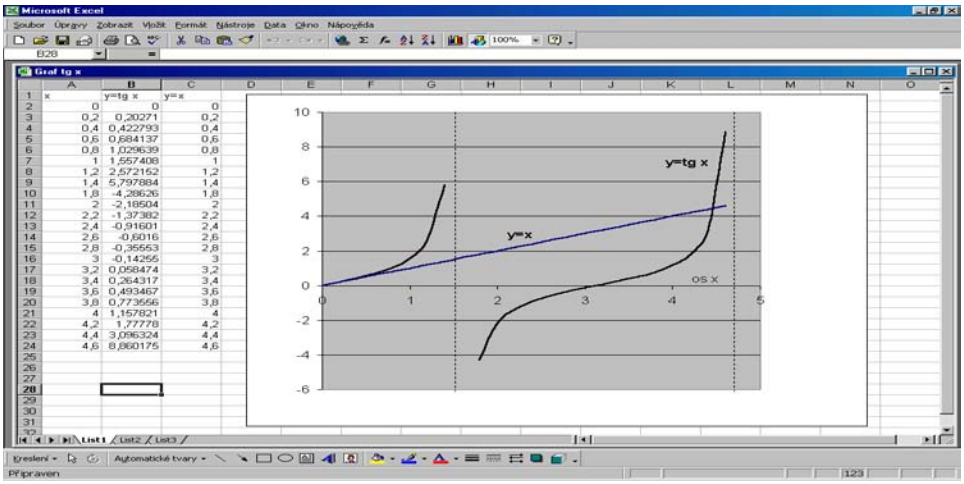


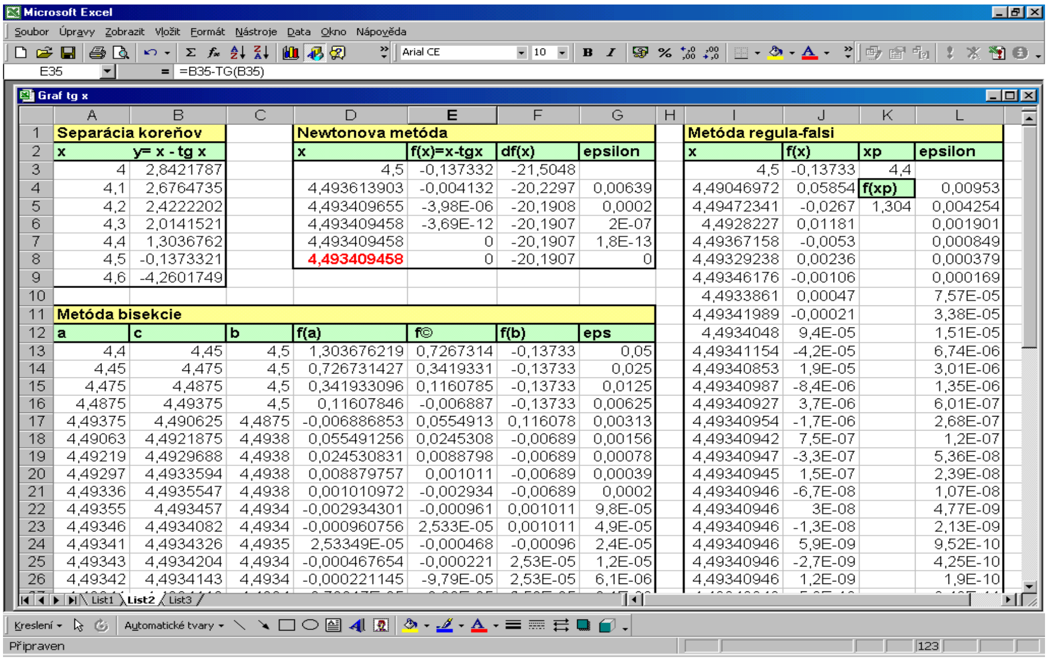
Fig.2 Graphical solution in MS Excel environment

MS Excel utilisation with a numerical solution to the equation (4) by the help of several numerical methods leads us very fast to the result with a required accuracy (fig.3). The use of a numerical method is always preceded by roots separation, i.e. a determination of such intervals $\langle a,b \rangle$, which contain only one root, thus it is valid $f(a).f(b)\langle 0$. For the roots separation of the equation (4) in Excel we will use its different shape : $x - tgx = 0$ (see fig. 3). To the plots A3 – A9 of column A we will write down the values of an independent variable x (we have already taken to consideration that we were searching for the smallest positive root), to the plot B3 we write the formula in the shape „=A3-TG(A3)“. The root of the equation (4) lies within the interval $\langle 4{,}4;4{,}5 \rangle$, because it holds $f(x) = x - tg\,x$, $f(4{,}4).\,f(4{,}5) = 1{,}30367.(-0{,}1373) < 0$. As we can see in fig. 3, the fastest possible way how to obtain an accurate root of the equation (4) is Newton's method of tangents. Using this method we will calculate $f'(x) = 1 - \frac{1}{\cos^2 x}$, $f''(x) = -\frac{2\sin x}{\cos^3 x}$, $f''(4{,}4) = -65{,}562928$, $f''(4{,}5) = -208{,}724911$.

Because $f(4{,}5).\,f''(4{,}5) = (-0{,}13733).(-208{,}72491) > 0$, as the first approximate value of the root we will take $b = 4{,}5$, i.e. we will draw a tangent at the point $B[4{,}4;1{,}303]$ and using the relation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{5}$$

we will gradually obtain more accurate values of the root. If solving in MS Excel, it means that for example to the cell D3 we will write the numeral 4.5; to the cell E3 the function $f(x)$, that is the formula „=D3-TG(D3)“, to the cell F3 $f'(x)$, it means the formula „=1-1/(COS(D3))^2“ and to the cell D4 an iterative formula (5) in the shape „=D3-E3/F3“, because Excel uses references to the cell. To the cells E4 and F4 we will copy the values $f(x)$ and $f'(x)$, to the cell G4 we will write the relation of accuracy, i.e. „=ABS(D4-D3). The root of the equation (4) is being specified in column D, this is proved by the values of epsilon in column G. For example, in the plot D6, the root of the equation (4) has the accuracy $\varepsilon = 2.10^{-7}$.



	A	B	C	D	E	F	G	H	I	J	K	L
1	Separácia koreňov			Newtonova metóda					Metóda regula-falsi			
2	x	y= x - tg x		x	f(x)=x-tgx	df(x)	epsilon		x	f(x)	xp	epsilon
3	4	2,8421787		4,5	-0,137332	-21,5048			4,5	-0,13733	4,4	
4	4,1	2,6764735		4,493613903	-0,004132	-20,2297	0,00639		4,49046972	0,05854	f(xp)	0,00953
5	4,2	2,4222202		4,493409655	-3,98E-06	-20,1908	0,0002		4,49472341	-0,0267	1,304	0,004254
6	4,3	2,0141521		4,493409458	-3,69E-12	-20,1907	2E-07		4,4928227	0,01181		0,001901
7	4,4	1,3036762		4,493409458	0	-20,1907	1,8E-13		4,49367158	-0,0053		0,000849
8	4,5	-0,1373321		4,493409458	0	-20,1907	0		4,49329238	0,00236		0,000379
9	4,6	-4,2601749							4,49346176	-0,00106		0,000169
10									4,4933861	0,00047		7,57E-05
11	Metóda bisekcie								4,49341989	-0,00021		3,38E-05
12	a	c	b	f(a)	f©	f(b)	eps		4,4934048	9,4E-05		1,51E-05
13	4,4	4,45	4,5	1,303676219	0,7267314	-0,13733	0,05		4,49341154	-4,2E-05		6,74E-06
14	4,45	4,475	4,5	0,726731427	0,3419331	-0,13733	0,025		4,49340853	1,9E-05		3,01E-06
15	4,475	4,4875	4,5	0,341933096	0,1160785	-0,13733	0,0125		4,49340987	-8,4E-06		1,35E-06
16	4,4875	4,49375	4,5	0,11607846	-0,006887	-0,13733	0,00625		4,49340927	3,7E-06		6,01E-07
17	4,49375	4,490625	4,4875	-0,006886853	0,0554913	0,116078	0,00313		4,49340954	-1,7E-06		2,68E-07
18	4,49063	4,4921875	4,4938	0,055491256	0,0245308	-0,00689	0,00156		4,49340942	7,5E-07		1,2E-07
19	4,49219	4,4929688	4,4938	0,024530831	0,0088798	-0,00689	0,00078		4,49340947	-3,3E-07		5,36E-08
20	4,49297	4,4933594	4,4938	0,008879757	0,001011	-0,00689	0,00039		4,49340945	1,5E-07		2,39E-08
21	4,49336	4,4935547	4,4938	0,001010972	-0,002934	-0,00689	0,0002		4,49340946	-6,7E-08		1,07E-08
22	4,49355	4,493457	4,4934	-0,002934301	-0,000961	0,001011	9,8E-05		4,49340946	3E-08		4,77E-09
23	4,49346	4,4934082	4,4934	-0,000960756	2,533E-05	0,001011	4,9E-05		4,49340946	-1,3E-08		2,13E-09
24	4,49341	4,4934326	4,4935	2,53349E-05	-0,000468	-0,00096	2,4E-05		4,49340946	5,9E-09		9,52E-10
25	4,49343	4,4934204	4,4934	-0,000467654	-0,000221	2,53E-05	1,2E-05		4,49340946	-2,7E-09		4,25E-10
26	4,49342	4,4934143	4,4934	-0,000221145	-9,79E-05	2,53E-05	6,1E-06		4,49340946	1,2E-09		1,9E-10

Fig.3 *Numerical solution to the non-linear equation* $tg\, x = x$ *by means of MS Excel*

Conclusion

Program products are also effectively utilised with strengthening of the mathematics applicable character as well as with a support of interdisciplinary relations.

Literature

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